Problem 2.17 ECEN 3613

$f = 300\,MHz$

Loss less

$Z_0 = 50\Omega$

air spaced ($\mu_r = 1$, $\varepsilon_r = 1$)

length $= 2.5\,m$

$Z_L = (40 + j20)\Omega$

$$Z_{in}(-l) = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

$$u_p = f\lambda = \frac{\omega}{\beta}$$

$$c = \frac{\omega}{\beta} \Rightarrow \beta = \frac{2\pi f}{c} = \frac{2\pi(3\times10^8)}{3\times10^8} = 2\pi \Rightarrow \beta l = 2\pi\left(\frac{5}{2}\right) = 5\pi$$

Hard way:

$$Z(-2.5) = 50\left[\frac{40 + j20 + j50\tan(5\pi)}{50 + j(40+j20)\tan(5\pi)}\right] \quad \tan(5\pi) = \tan(\pi) = 0$$

$$Z_{in}(-2.5) = 50\left[\frac{40+j20}{50}\right]$$

$$Z_{in}(-2.5) = \boxed{40 + 20j}$$

easy way:

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^8} = 1\,m \Rightarrow \frac{\lambda}{2} = \frac{1}{2}m \Rightarrow l = 5\left(\frac{\lambda}{2}\right)$$

$Z_{in}(z)$ repeats every $\frac{\lambda}{2}$ $\therefore Z_{in}\left(-\frac{5}{2}\lambda\right) = Z_L = \boxed{40 + j20}$

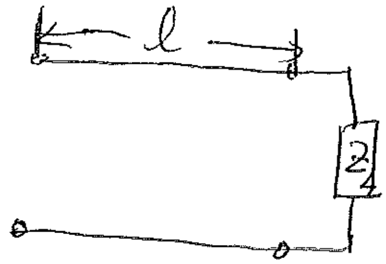
Problem 2.18 ECEN 3613

Loss less

$l = 0.35\lambda$

$Z_0 = 100\Omega$

$Z_L = (60 + j30)\Omega$



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j30 - 100}{60 + j30 + 100} = \frac{-40 + j30}{160 + j30} = \frac{50\angle 143.13^\circ}{162.79\angle 10.62}$$

$\boxed{\Gamma = 0.3071\angle 132.51^\circ}$ (or $-0.2075 + j\,0.2264$)

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.3071}{1 - 0.3071} = \frac{1.3071}{0.6929}$$

$\boxed{S = 1.886}$

$\beta = \frac{2\pi}{\lambda}$ $\quad l = 0.35\lambda \Rightarrow \beta l = 2\pi(0.35) = 126^\circ$

$$Z_{in}(-l) = Z_0\left[\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right] = 100\left[\frac{60 + j30 + j100\tan(126^\circ)}{100 + j(60 + j30)\tan(126^\circ)}\right]$$

$$= 100\left(\frac{60 - j107.64}{141.29 - j82.58}\right) = 100\,\frac{123.23\angle -60.86^\circ}{163.65\angle -30.31^\circ}$$

$= 75.30\angle -30.55$

$Z_{in}(-l) = \boxed{64.85 - j38.27\Omega}$

Problem 2.19 ECEN 3613

Lossless

$Z_L = 0$ (short Circuit)

$l = \frac{\lambda}{4}$

$$Z_{in}(-l) = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

$\beta = \frac{2\pi}{\lambda}$ $l = \frac{\lambda}{4}$ $\Rightarrow \beta l = \frac{\pi}{2}$ $\tan\frac{\pi}{2} = \infty$

$$\Rightarrow Z_{in}(-\tfrac{\lambda}{4}) = Z_0 \left[\frac{\frac{Z_L}{\infty} + jZ_0}{\frac{Z_0}{\infty} + jZ_L} \right] = \frac{Z_0^2}{Z_L}$$

$$Z_{in}^{sc}(-\tfrac{\lambda}{4}) = \frac{Z_0^2}{Z_L}$$

But $Z_L = 0 \Rightarrow Z_{in}^{sc} = \frac{Z_0^2}{0} = \infty$

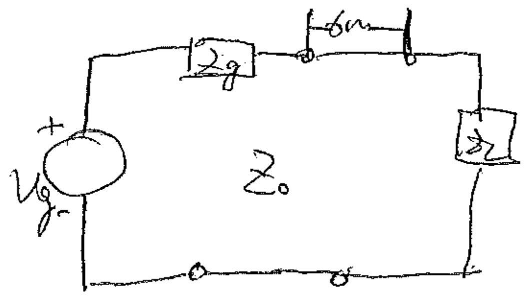
$\therefore Z_{in}^{sc}(-\frac{\lambda}{4}) =$ open circuit

Problem 2.22 ECEN 3613

$l = 6\,m$ $\qquad \omega = 8\pi\times10^7 \; (f = 40\,MHz)$

$Z_0 = 150\,\Omega$ $\qquad \tilde{V}_g = 5\angle -30^\circ$ (cos ref)

loss less



$V_g(t) = 5\cos(8\pi\times10^7 t - 30^\circ)\,V$

$Z_g = 150\,\Omega$

$\varepsilon_r = 2.25$

$Z_L = 150 - j150\,\Omega$

(a) $u_p = \lambda f = \frac{\omega}{\beta}$ $\qquad u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = \frac{c}{\sqrt{\varepsilon_r}}$

$\lambda = \frac{c}{f\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{2.25}\,4\times10^7} =$

$\boxed{\lambda = 5m}$ $\Rightarrow \beta = \frac{\omega}{u_p} = \frac{2\pi}{\lambda} = \frac{2\pi}{5}$

(b) $\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 - j50 - 150}{150 - j50 + 150} = \frac{-j50}{300 - j50} = \frac{50\angle -90^\circ}{304.14\angle -9.46^\circ}$

$= 0.1644\angle -80.54$

$\boxed{\therefore \Gamma = 0.1644\angle -80.54}$

(c) $Z_{in}(-l) = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$ $\qquad \beta l = \left(\frac{2\pi}{5}\right)(6) = \frac{12\pi}{5} = 432^\circ$ or 72°

$Z_{in}(-6) = 150\left[\frac{150 + j411.65}{303.88 + j461.65}\right] = 150\,\frac{438.13\angle 69.98^\circ}{552.69\angle 56.65^\circ} = 118.81\angle 13.33^\circ$

$Z_{in}(-6) = \boxed{115.71 + j27.42\,\Omega}$

(d) $\tilde{V}_i = \frac{Z_{in}}{Z_g + Z_{in}}\tilde{V}_g = \left(\frac{118.81\angle 13.33}{150 + 115.71 + j27.42}\right)(5\angle -30) = \frac{594.55\angle -16.67^\circ}{267.12\angle 5.89^\circ}$

$= 2.226\angle -22.56^\circ$

$\boxed{\tilde{V}_i = 2.226\angle -22.56\,V}$

(e) $V_i(t) = 2.226\cos(8\pi\times10^7 t - 22.56^\circ)\,V$

Problem 2.25 ECEN 3613

loss less

Z_L is shortcircuit ($Z_L = 0$)

desire opencircuit at input

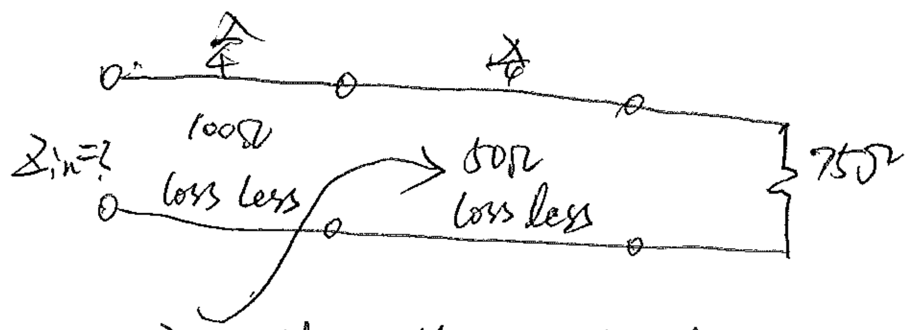
line repeats every $\frac{\lambda}{2}$, "inverts" every $\frac{\lambda}{4}$

so a quarter wave line (or odd multiples of $\frac{\lambda}{4}$) would appear as open.

i.e., $l = \frac{\lambda}{4} + n\frac{\lambda}{2}$, $n = 0, 1, 2, \ldots$

or $l = n\frac{\lambda}{4}$, n odd

Problem 2.27 ECEN 3613



Zinmid will be the load for the 100Ω section

$\beta = \frac{2\pi}{\lambda} \Rightarrow \beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$ so $\tan\beta l = \infty$ $\therefore Z_{in} = \frac{Z_0^2}{Z_L}$

$Z_{in\,mid} = \frac{(50)^2}{75} = \boxed{33.33\Omega}$

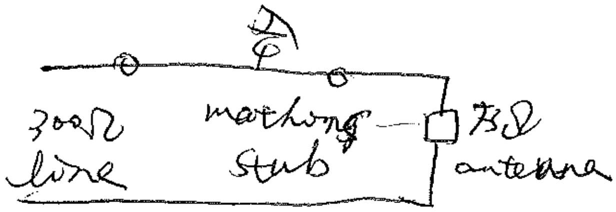
$Z_{in} = \frac{(100)^2}{Z_{in\,mid}} = \frac{(100)^2}{33.33} = 300$

$Z_{in} = \boxed{300\Omega}$

Problem 2.28 ECEN 3613

$f = 100\,MHz\ (1\times10^{8}\,Hz)$

$300\,\Omega$ line (assumed loss less)



$Z_L = 73\,\Omega$ (antenna)

(a) $Z_{in\,stub} = Z_{0\,stub}^2 / Z_L$, length $= \frac{\lambda}{4}$

We want $Z_{in\,stub} = 300\,\Omega \Rightarrow Z_{0\,stub} = \sqrt{(300)(73)} = \boxed{148\,\Omega}$

(b) Two-wire, $d = 2.5\,cm$ $\epsilon_r = 2.6$

$u_p = f\lambda = \frac{c}{\sqrt{\epsilon_r}}$ $\lambda = \frac{c}{\sqrt{\epsilon_r}\,f} = \frac{3\times10^8}{\sqrt{2.6}\,1\times10^8} = \frac{3}{\sqrt{2.6}} = 1.8605\,m$

$\frac{\lambda}{4} = \boxed{0.4651\,m}$

Ref Table 2.2 P55:

Two wire, loss less ($R' = 0$ $G' = 0$) a = wire radius

$Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln\left[\frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\right]$ d = .. separation

let $g = \frac{d}{2a}$ $f = Z_0\sqrt{\epsilon_r}/120$

$\Rightarrow f = \ln[g + \sqrt{g^2 - 1}]$ $g + \sqrt{g^2-1} = e^f$ $\sqrt{g^2-1} = e^f - g$

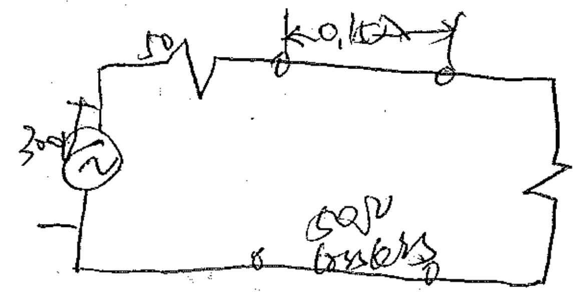
$g^2 - 1 = e^{2f} - 2e^f g + g^2 \Rightarrow g = \frac{e^{2f}+1}{2e^f}$

then $\frac{d}{2a} = \frac{e^{\frac{Z_0\sqrt{\epsilon_r}}{60}} + 1}{2e^{\frac{Z_0\sqrt{\epsilon_r}}{120}}} = \frac{54.377}{14.612}$

$\frac{d}{2a} = 3.7214 \Rightarrow \boxed{a = 3.36\,mm}$

2.31 ECEN 3613

$\tilde{V}_g = 300V$ lossless line
$Z_g = 50\Omega$ w/ $Z_0 = 50\Omega$
$Z_L = 75\Omega$ $l = 0.15\lambda$



note: matched at source ($\Gamma_{source} = 0$)

(a) $Z_{in}(-l) = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right)$

$\beta = \frac{2\pi}{\lambda}$, $\beta l = \frac{2\pi}{\lambda} \times 0.15 = 0.3\pi \ (54°)$

$Z_{in}(-l) = 50 \left(\frac{75 + j50\tan 54°}{50 + j75\tan 54°} \right) = 50 \left(\frac{1.5 + j1.3764}{1 + j2.0646} \right)$

$= 50 \frac{2.0358 \angle 42.54°}{2.2940 \angle 64.15°}$

$Z_{in}(-l) = 44.37 \angle -21.62° = \boxed{41.25 - j16.35\,\Omega}$

(b) $\tilde{V}_i = \frac{Z_{in}}{Z_{in} + Z_g} \tilde{V}_g = \frac{44.37 \angle -21.62°}{41.25 - j16.35 + 50} (300)$

$= \frac{13311 \angle -21.62}{92.70 \angle -10.16} = \boxed{143.6 \angle -11.46° \ V}$

$\tilde{I}_i = \frac{\tilde{V}_i}{Z_{in}} = \frac{143.6 \angle -11.46°}{44.37 \angle -21.62°} = \boxed{3.236 \angle 10.16°}$

(c) $P_{in} = \frac{1}{2} Re[\tilde{V}_i \tilde{I}_0^*] = \frac{1}{2} Re[(143.6 \angle -11.46) 3.236 \angle -10.16°]$

$= \frac{1}{2} Re[464.7 \angle -21.62] = \frac{1}{2}(464.7)\cos(-21.62) = \boxed{216W}$

(d) $\tilde{V}_L = \tilde{V}(z)|_{z=0} = V_0^+(1+\Gamma)$, $\tilde{I}_L = \tilde{I}(z)|_{z=0} = \frac{V_0^+}{Z_0}(1-\Gamma)$

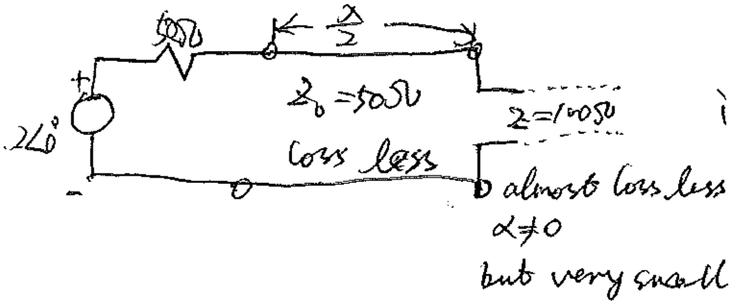
$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75-50}{75+50} = \frac{25}{125} = \frac{1}{5}$, $V_0^+ = \underbrace{\frac{Z_{in}}{Z_g + Z_{in}} \tilde{V}_g}_{\tilde{V}_i} \frac{1}{\underbrace{e^{j\beta l}}_{\cos\beta l + j\sin\beta l} + \underbrace{\Gamma e^{-j\beta l}}_{\Gamma\cos\beta l - j\Gamma\sin\beta l}}$

$V_0^+ = 143.6 \angle -11.46 \left(\frac{1}{(1+\frac{1}{5})\cos 54° + j(1-\frac{1}{5})\sin 54°} \right)$

$V_0^+ = \frac{143.6 \angle -11.46°}{0.9573 \angle 42.54°} = 150 \angle -54° \Rightarrow \tilde{V}_L = (1 + \frac{1}{5}) 150 \angle -54$

$\tilde{V}_L = 180 \angle -54°$

problem 2.33 ECEN 3613



infinitely long ($l \gg \lambda$ so that at end $e^{-\alpha z} \simeq 0$!)

i.e., no reflections because no voltage to reflect!)

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \text{ (or } 180°)$$

so $\tan \beta l = 0$ $\quad e^{j\pi} = -1, \quad e^{-j\pi} = -1$

$$\Gamma = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3}, \quad Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = 50\left[\frac{100}{50}\right] = 100$$

$$V_0^+ = \left(\frac{Z_{in}}{Z_g + Z_{in}}\right)\tilde{V}_g\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \left(\frac{100}{50+100}\right)2\angle 0°\left(\frac{1}{-1+\frac{1}{3}(-1)}\right) = -1 = 1\angle 180°$$

$$\tilde{V}_i = \frac{Z_{in}}{Z_g + Z_{in}}\tilde{V}_g = \frac{100}{50+100}2\angle 0° = \frac{4}{3}\angle 0° \qquad \tilde{I}_i = \frac{\tilde{V}_i}{Z_{in}} = \frac{4}{300}\angle 0°$$

$$P_i = \frac{1}{2}\text{Re}[\tilde{V}_i \tilde{I}_i^*] = \frac{1}{2}\text{Re}\left[\frac{4}{3}\angle 0° \frac{4}{300}\angle 0°\right] = \frac{8}{900}W = 8.89mW\left[\frac{80}{9}mW\right]$$

$$P_W^i = \frac{|V_0^+|^2}{2Z_0} = \frac{1}{2(50)} = \frac{1}{100}W = 100mW\left[\frac{90}{9}mW\right]$$

$$P_W^r = -|\Gamma|^2 P_W^i = -\left(\tfrac{1}{3}\right)^2 10mW = -\frac{10}{9}mW$$

$$P_W^t = P_i$$

P_i goes to the load as well (loss less line)

Power check:

$$\boxed{\begin{array}{ll} \text{incident}: & \frac{90}{9}mW \\ \text{reflected}: & -\frac{10}{9}mW \\ \text{delivered}: & \frac{80}{9}mW \end{array}}$$

checks $\frac{90}{9} - \frac{10}{9} = \frac{80}{9}$!

Problem 3.33   ECEN 3613

$\vec{\nabla} T = e^{-2z}\hat{a}_z$, $T = 10$ at $z = 0$

$\vec{\nabla} T = \frac{\partial T}{\partial z}\hat{a}_z = \frac{dT}{dz}\hat{a}_z$ (Cartesian or Cylindrical]

because only one variable

$\frac{dT}{dz} = e^{-2z}$ $dT = e^{-2z}dz$ $\int dT = \int e^{-2z}dz$

$T = \frac{e^{-2z}}{-2} + k$ (k is the constant of integration)

at $z = 0$, $T = 0$

$\Rightarrow 10 = -\frac{1}{2} + k \Rightarrow k = 10.5$

$T(z) = 10.5 - 0.5e^{-2z}$

(Text answer is for $\vec{\nabla} T = e^{3z}\hat{a}_z$)

Problem 3.38 ECEN 3613

(a) $\vec{E}$ points along $\hat{a}_R$ $\Rightarrow \vec{E} = E_R \hat{a}_R$

(b) $|\vec{E}|$ is a function of R only $\Rightarrow |\vec{E}| = E_R = E_R(R)$

a function of R, not times R!

(c) $\vec{E}|_{origin} = 0 \Rightarrow E_R(R)|_{R=0} = 0$

(d) $\vec{\nabla} \cdot \vec{E} = 12$ everywhere $\Rightarrow \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 E_R) = 12 \Rightarrow \frac{d}{dR}(R^2 E_R) = 12R^2$

$d(R^2 E_R) = 12R^2 dR$

$R^2 E_R(R) = 4R^3 + R$

for $R=0$, $E_R(0)$ $R^n = 0 \Rightarrow R = 0$

$R^2 E_R(R) = 4R^3$

$E_R(R) = 4R$

$\vec{E} = 4R\,\hat{a}_R$